A PDP-9 computer program for on-line calculation of mean values, variances, and amplitude distribution

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INTRODUCTION

The extrapolation from a plot of the inverse variance ($1/\sigma^2$) vs a system parameter of interest can be used to indicate the onset of instability for certain systems. This method has been discussed theoretically by Akcasu (1961). Important system parameters for variance analysis in nuclear power reactors are, e.g., power, subcooling, and pressure. This type of experimental analysis has been applied to the Halden reactor (Eurola, 1964) and the Frigg heat-transfer loop (Nylund et al, 1967). An on-line variance computer program is developed for measurements at the Marviken nuclear power station.

This program belongs to a software package written for a PDP-9 application to on-line measurements at Marviken (Bliselius & Schuch, 1968). The program is written in Symbolic Assembler (Basic) language, a PDP-9 computer-oriented language. The calculations are carried out in unsigned integer arithmetic in order to obtain a short execution time and minimum memory space of the program.

The calculation of mean values and variances is based on the multichannel analyzer idea, which is used in conventional pulse height analysis. Briefly, this means an on-line software sorting of the amplitude samples and up-dating of the "spectrum" or the "amplitude distribution" (a longer but more correct term is the "probability density of the amplitude distribution"). Then the mean value and variance calculation are easily performed based on the accumulated data stored as distributions, which is shown in the theory section.

This program is designed to take care of a maximum six signals simultaneously at 10-bit resolution or 1,024 sampling or quantizing levels. This gives a capacity of $6 \cdot 1{,}024 = 6{,}144$ channels, if we compare it to a multichannel analyzer. It is also possible to perform on-line measurements at 12 signals at 9-bit resolution. Thus, 6k core memory space is occupied as data area and the 2k remainder is reserved for program. From the detailed listing in Table 1, we find a program space requirement of about 700 words.

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THEORY

Mean Value Calculation

Let the m quantizing or sampling levels be denoted $x_1, x_2, \cdots, x_m$. The first channel, x_1, corresponds to −10 V and x_m to +10 V. We call the number of events in each channel of the distribution $f_1, f_2, \cdots, f_m$. By definition, we obtain as mean value, $\bar{x}$:

$$f_1 x_1 + f_2 x_2 + \cdots + f_m x_m$$

$$= \bar{x}(f_1 + f_2 + \cdots + f_m)$$

and

$$\bar{x} = \frac{\sum_{i=1}^{m} f_i x_i}{\sum_{i=1}^{m} f_i} \tag{1}$$

The distribution is divided into $m = 2^n$ channels and we denote these as follows:

$$-(2^{n-1}-1), -(2^{n-1}-2), \cdots,$$
$$-1, -0, 0, +1, \cdots,$$
$$(2^{n-1}-2), (2^{n-1}-1) \tag{2}$$

We use the notation "−0" as the standard subroutine "Decimal Integer Print" works with one's complement arithmetic.

In order to avoid calculating with negative numbers, we bias all x_i values (channel numbers) with b. Eq. 1 can

Table 1
Listing of Subroutines

Type of Subroutine	Number of Words (Decimal)	Comments
Main program with identification and service of interrupts	40	Page A:2
Real time clock and multiplexer service	43	
Initialization of sorting routine	14	
Sorting of samples	65	
Teletype routine	5	PDP-9 standard
Unsigned multiplication, single precision routine	29	routines (BASIC)
Unsigned division, single precision routine	33	
Teletype output package	184	The developed
Decimal integer print	53	routines are based
Mean value calculation	66	on the theory in
Variance calculation	70	Section 2
Result printout routine	65	
	Sum: 667 Words	
Funny format (FF) loader	119	PDP-9 standard
RIM loader	14	routines (BASIC)
Data area (6k)	6144	
	Total: 6944 Words	

Main Program and Interrupt Service Routines

```
ON-LINE VARIANSPROGRAM    9  DEC  1968
/P A BLISELIUS
160/
BEGIN,         JMS  INIT1        /INITIERA SORTERINGSRUTIN
               DZM  LOOP1        /NOLLSTAELL WAITING LOOP
               DZM  LOOP2
               CAF               /CLEAR ALL FLAGS
               LAC  DELTAT       /FOERSTAELL KLOCKA
               CMA
               TAD  (1
               DAC  7
               DAC  LDELTA
               CLON
               ION
               ISZ  LOOP1        /WAITING LOOP
               JMP  .-1
               ISZ  LOOP2
               JMP  .-3
LDELTA,        0
DELTAT,        5                 /5 KLOCKPULSER GER 0.1 S SAMPLINGS INTERVAL
LOOP1,         0
LOOP2,         0
INIT1 = 340
```

Table 1 (Continued)

```
/INTERRUPT SERVICE RUTIN
SERVIS,         DAC  AC             /LAGRA AC:S INNEHALL
                CLA
                CLSF                /SKIP OM CLOCK FLAG = 1
                SKP
                JMS  CLOCK          /CLOCK RUTIN
                LAS
                SPA                 /SKIP OM BIT Ø = Ø
                JMS  PRINT          /BERAEKNINGS- OCH
                KSF                 /SKIP OM TELETYPE/KBD FLAG = 1
                SKP
                JMS  PRINT
                CAF                 /NOLLSTAELL ALLA FLAGGOR
                LAC  Ø              /ATERSTAELL LINK
                RAL
                LAC  AC             /ATERSTAELL ACKUMULATOR
                ION
                JMP  I   Ø          /ATERGA TILL WAITING LOOP
AC,
PRINT = 164Ø
CLOCK = 3ØØ
1/
                JMP  SERVIS         /HOPPA TILL INTERRUPT SERVICE RUTIN
START BEGIN
```

then be written:

$$\overline{x}_{cal} = \overline{x}_{true} + b = \frac{\sum_{i=1}^{m} f_i(x_i + b)}{\sum_{i=1}^{m} f_i} \quad (3)$$

We can now treat the channel numbers as a set $(x_i + b)$ of integers:

$$(x_i + b) = (0, 1, 2, \cdots, 2^n - 1)$$

This application uses n = 10 and n = 9 and we obtain 1,024 channels and 512 channels of the distribution, respectively.

The channel width, w, of the distribution is:

$$w = \frac{20}{2^n} \text{ V/channel} \quad (4)$$

w = 0.01953 V/channel for n = 10 and w = 0.03906 V/channel for n = 9.

The printout of mean values has two columns; one gives the channel number according to Relation 2 and the other one gives the mean in millivolts based on Eq. 4.

Variance Calculation

For the variance calculation we use the following definition:

$$\sigma^2 = \overline{x^2} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \overline{x})^2 \quad (5)$$

Assume again $x_1, x_2, \cdots, x_m$ as different channels of the distribution and $f_1, f_2, \cdots, f_m$ as the number of events in each channel, respectively. We can then write:

$$f_1(x_1 - \overline{x})^2 + f_2(x_2 - \overline{x})^2 + \cdots$$
$$+ f_m(x_m - \overline{x})^2$$
$$= \overline{x^2}(f_1 + f_2 + \cdots + f_m)$$

and the variance or mean square value is:

$$\overline{x^2} = \frac{\sum_{i=1}^{m} f_i(x_i - \overline{x})^2}{\sum_{i=1}^{m} f_i} \quad (6)$$

From the calculation in Eq. 3 we get a biased mean value $\overline{x}_{cal} = M$, which is related to the channel set $(x_i + b) = (0, 1, \cdots, 2^n - 1)$. The differences $d_i = x_i - \overline{x}$ of Eq. 6 are computed with biased parameters:

$$(d_i) = (x_i) - \overline{x}_{true}$$
$$= (x_i + b) - (\overline{x}_{true} + b)$$
$$= (x_i + b) - M$$

We observe that the absolute amount of d_i can be written as a set as follows:

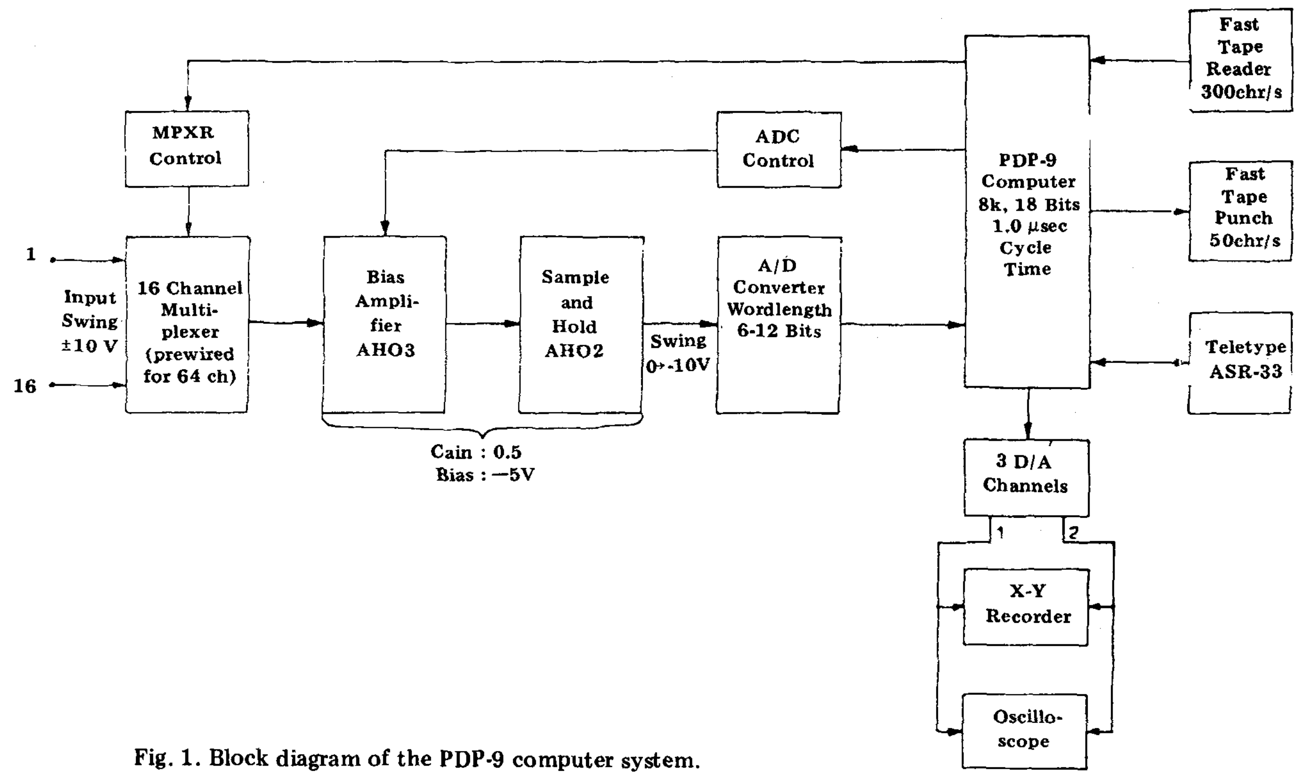

Fig. 1. Block diagram of the PDP-9 computer system.

Table 2
System Conversion Characteristics

Word Length Number of Bits	Maximum Quantizing Error[x] in Percent	A/D Conversion Time in Msec	Total Conversion Time (MPXR, AHO3, AHO2, and ADC) in Msec
6	±1.6	9.0	21.0
7	±0.8	10.5	22.5
8	±0.4	12.0	24.0
9	±0.2	13.5	25.5
10	±0.1	18.0	30.0
11	±0.05	25.0	37.0
12	±0.025	35.0	47.0

[x]±0.5 LSB (least significant bit).

$$(d_i) = [M, M-1, M-2, \circ\circ\circ, 1, 0, 1, \bullet\bullet\bullet, (2^n-1-M)]$$

Now we build up the variance computation algorithm by squaring consecutive positive integers as $M \geqslant 0$. The printout of variances is given in channel number squared.

PDP-9 APPLICATION TO MARVIKEN MEASUREMENTS

The PDP-9 equipment available for the Marviken application is shown in Fig. 1. A swing of x 10 V is allowed in the input signals. The — conversion time and quantizing error are found in Table 2.

The printout of means and variances is made by the teleprinter. The possibility of using two D/A channels for displaying a distribution function is considered. Such a display subroutine is included in the on-line program for correlation functions (Bliselius & Schuch, 1968). Figure 2 demonstrates the time diagram for data collecting and reduction for 12 input signals. Table 3 indicates the total effective time (hardware + software) used for sampling and sorting during a sampling interval. This means a possible total sampling rate of

$$f_{max} = \frac{6}{650 \circ 10^{-6}} \approx 9{,}200\ \text{Hz}$$

at 12-bit A/D conversion.

The sampling intervals are generated by the PDP-9 real-time clock. This operates maximally at 50 Hz and gives a minimum sampling interval of 20 msec.

Table 3
Total Time Used for Sampling and Sorting

Number of Input Signals	Time in μsec at Different Word Length of the A/D Conversion: 6 Bits	10 Bits	12 Bits
6	500	550	650
8	650	740	860
12	980	1080	1280

From Table 3 we conclude that we have a good time margin during a sampling interval. Actually we use maximum

$$\frac{1{,}280 \circ 10^{-6}}{20 \circ 10^{-3}} = 6.4\%$$

of the time available during a 20-msec sampling interval. Compare with Fig. 2.

The signals from the Marviken plant (maximum ±10 V) are connected to the multiplexer channels sequentially. It is suggested that the A/D converter operates at 12-bit word length, even if the program only works with the 10 MSB (most significant bits) at 6 signals and the 9 MSB at 12 signals. Thus we obtain 6 amplitude distributions with 1,024 channels (intervals) or 12 distributions with 512 channels, respectively.

This is illustrated in Fig. 3. The resolution is given by Eq. 4. The number of samples per signal is also counted by PDP-9.

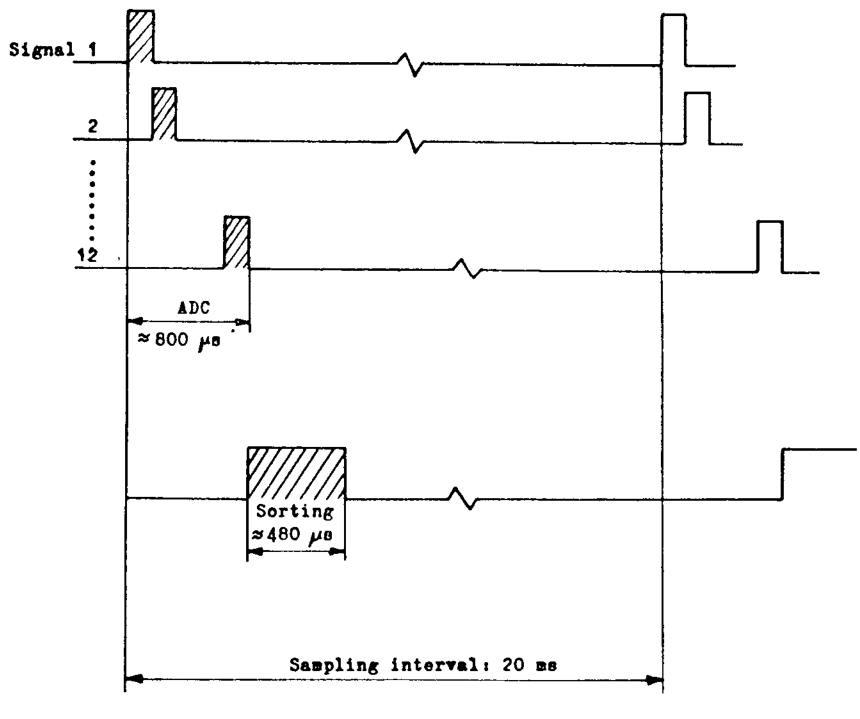

Fig. 2. Time diagram for data collecting and data reduction.

DESCRIPTION OF THE PROGRAM (ACCORDING TO DECUS RECOMMENDATIONS)

Program Name
On-line variance program.

Date
Latest edition is of February 19, 1969.

Purpose
See introduction.

Programming Language
Symbolic Assembler, PDP-9 Basic.

Hardware Used
Basic PDP-9, A/D converter, and multiplexer (AFO1 unit); see Fig. 1.

Software Used
The following PDP-9 Basic standard routines are used: (1) unsigned multiplication, single precision; (2) unsigned division, single precision; (3) Teletype output package; and (4) decimal integer print.

Preparation of Input Data
If 6, 8, or 12 input signals are selected, all input data and parameters necessary are punched on the program (object) tape. Therefore, three versions of the program tape exist, Tapes 1, 2,

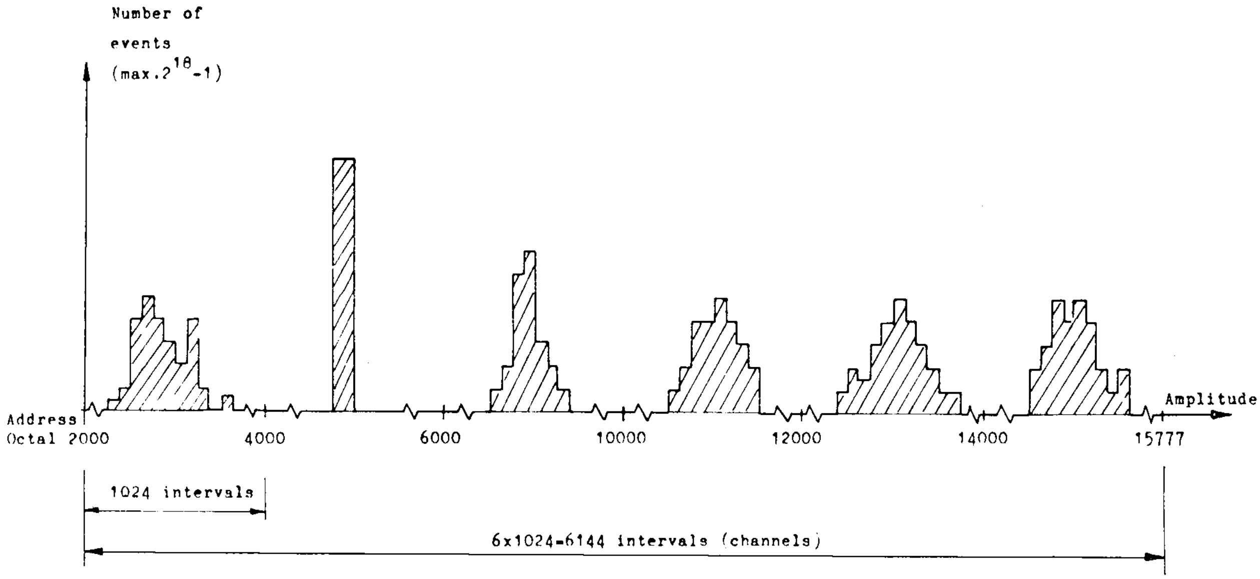

Fig. 3. Amplitude distribution functions stored in core memory (6 signals of 1024 intervals each).

Table 4

f_s (Hz)	T_{max} (Sec)
50	5242 ≈ 87 Min
10	26210 ≈ 435 Min

and 3. Only the sampling interval in octal is set manually in the address location 200 (DELTAT).

If no value is specified, the program works automatically with a 0.1-sec sampling interval. The parameter "ANKAN" determines the number (*octal*) of input signals. Changes can be made in Addresses 326, 444, and 173.

Operating Instructions

The instructions have the form of a flow diagram and are shown in Fig. 4. The program object tape is read in by the RIM and FF LOADERS. After read in the program is executed.

Restarting Addresses

ADS = 160 (data area is cleared).

ADS = 161 (no clearance; the data accumulation is continued).

Accumulator Switches (ACS)

BIT 0: UP = printout obtained; DOWN = no printout.

Glossary

DALTAT: Address 200—Load number of clock pulses (n). Sampling interval, $\Delta t = n \cdot 20$ msec.

ANKAN: Address 326, 444, 1733—Load number of input signals.

LOW: Address 446—Counter (double precision) for number of samples per signal.

HIGH: Address 447—Counter (double precision) for number of samples per signal.

MEDEL: Address 1574—Calculated biased mean value.

MV: Address 1576—Calculated true mean value.

KVOT: Address 1405—Calculated variance.

Method Description

See theory section.

Flow Diagram

Main flow diagram is shown in Fig. 5.

Program

The different subroutines used are listed in Table 1. As examples of routines, the main program and the interrupt service routine are shown in the same table.

Restrictions

The program can treat maximum $2^{18} - 1$ = 262,143 data per input signal. Maximum measuring time, T_{max}, is (f_s = sampling rate per signal). (See Table 4.)

$$T_{max} = 262{,}143 \cdot \Delta t = \frac{262{,}143}{f_s} \text{ sec}$$

Memory Required

The program needs 667_{10} words and is stored in the area $0 - 1740_8$. The data area requires 6k = 6144_{10} words and these are in the addresses 2000-15777. RIM and FF LOADERS are stored in the addresses 17573-17777 (133_{10} words).

Thus, the total program and data area require 6944_{10} words. There are about 1200_{10} words of the core memory in spare.

Time Required

Read in of the program tape requires: (1) about 15 sec, if RIM and FF LOADERS are stored in memory; and (2) about 45 sec, if the loaders are not in memory. Calculation and printout of means and variances require: (1) about 25 sec, for 8 signals; and (2) about

Table 5

KAN[a]	MV[b]		VAR[c]
ϕ	257	5ϕ19	1
1	258	5ϕ39	4
2	259	5ϕ58	9
3	26ϕ	5ϕ78	16
4	261	5ϕ97	25
5	262	5117	36

[a]*KAN = input signal or channel.*
[b]*MV = mean value.*
[c]*VAR = variance.*

Table 6

	Sample 1		Sample 2	
Signal	Accumulator	Channel Number	Accumulator	Channel Number
0	200000	256	201000	258
1	200000	256	202000	260
2	200000	256	203000	262
3	200000	256	204000	264
4	200000	256	205000	266
5	200000	256	206000	268

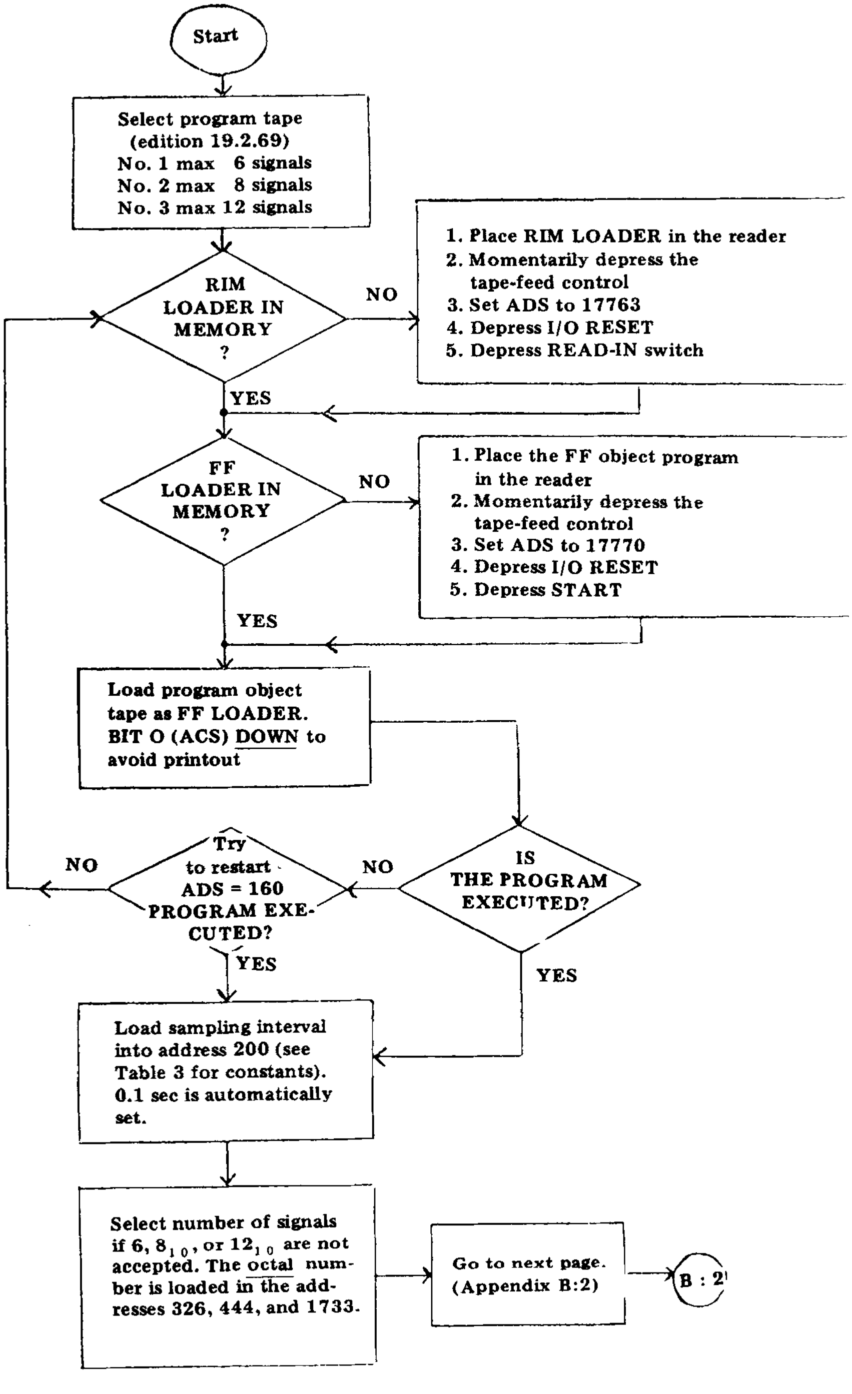

Fig. 4. Flow diagram for operating the PDP-9 computer at on-line variance measurements. (Continued on page 00.)

36 sec for 12 signals. This time is completely dominated by the Teletype printout speed.

Printouts

The example in Table 5 was obtained from a simulation of two samples per signal (6 signals and thus 10-bit resolution). The first column of the mean value gives this as "channel number" of the distribution and the second one is the mean value in millivolts. The variance has a dimension called, "channel number squared."

Other Remarks

Manual interrupt for on-line calculation and printout can be given by any key at the Teletype keyboard, when $f_s \leq 10$ Hz.

ACS BIT 0 can always be used.

PROGRAM TESTS BY SIMULATION

The resulting printout in Table 5 was based on the simulation in Table 6,

Table 7
Examples of Test Result Printouts

KAN	MV		VAR	
a) 0	0	0	0	
1	1	19	0	
2	2	39	0	5A
3	3	58	0	
4	4	78	0	
5	5	97	0	
b) 0	0	0	0	
1	−0	−19	0	
2	−1	−39	0	5B
3	−2	−58	0	
4	−3	−78	0	
5	−4	−97	0	
c) 0	128	5000	0	
1	128	5000	0	
2	128	5000	0	
3	128	5000	0	
4	128	5000	0	5C
5	128	5000	0	
6	128	5000	0	
7	128	5000	0	
d) 0	−127	−4999	0	
1	−127	−4999	0	
2	−127	−4999	0	
3	−127	−4999	0	
4	−127	−4999	0	5D
5	−127	−4999	0	
6	−127	−4999	0	
7	−127	−4999	0	
e) 0	−128	−5039	1	
1	−129	−5078	4	
2	−130	−5117	9	
3	−131	−5156	16	
4	−132	−5195	25	
5	−133	−5234	36	
6	−134	−5273	49	5E
7	−135	−5312	64	
8	−136	−5351	81	
9	−137	−5390	100	
:	−138	−5429	121	
;	−139		144	

where the digital value of 200000 corresponds to +5.000 V. The exact mean values of Samples 1 and 2 in Table 6 are thus given by the relation:

$$\bar{x} = 5000 + n \cdot \frac{200000}{1024}\ \mathrm{mV} \qquad (7)$$

$$n = 1, 2, \ldots, 6$$

The mean values in "channel numbers" are found as:

$$\bar{x} = 256 + n, \qquad n = 1, 2, \ldots, 6 \qquad (8)$$

The variance values are given by:

$$\sigma^2 = \frac{n^2 + n^2}{2} = n^2 \qquad (9)$$

$$n = 1, 2, \ldots, 6$$

In Table 7 (a) and (b), the results around zero channel numbers are shown. The one's complement arithmetic of the standard routine "decimal integer printer" gives both "0" and "−0" as a result. This routine prints also an extra space, when the result is zero.

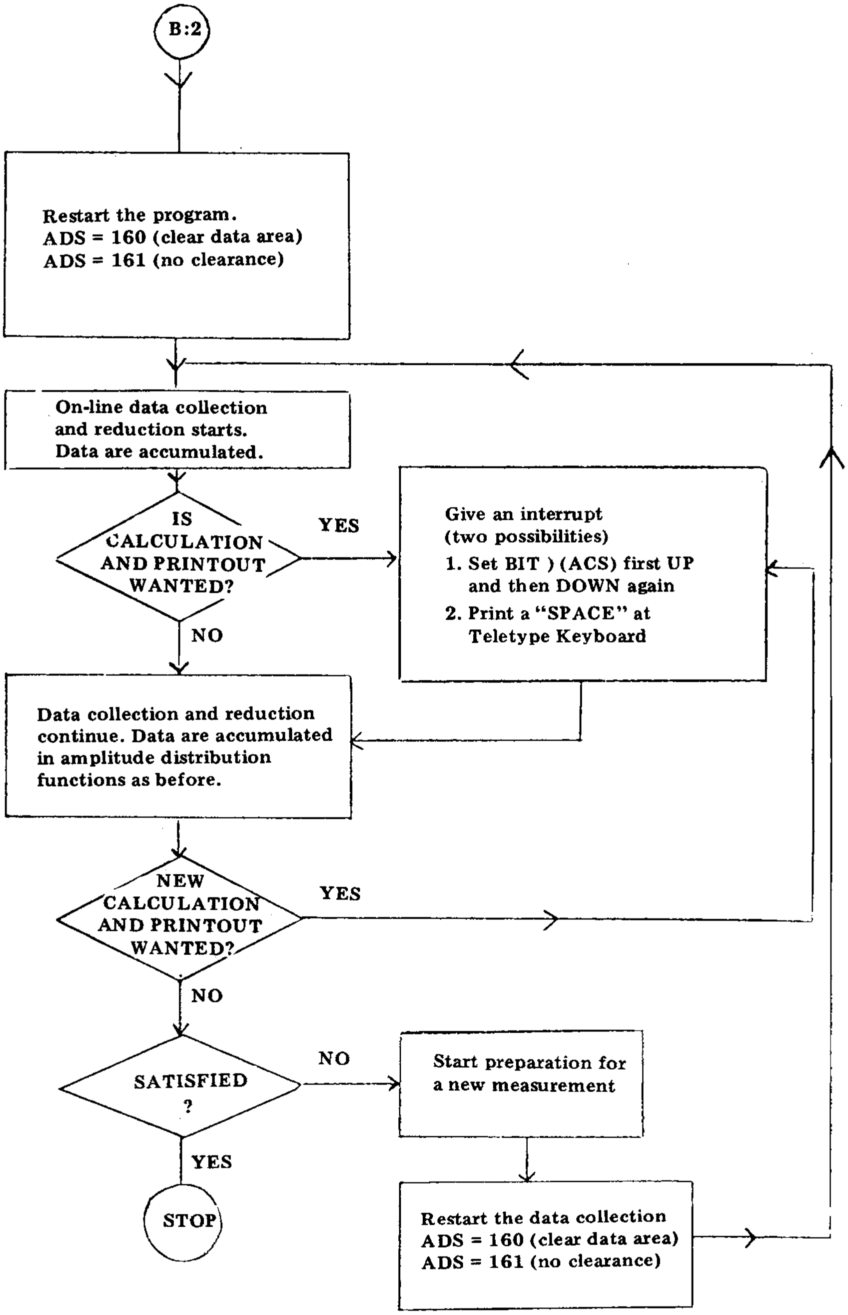

Fig. 4 (Cont.)

Table 8
Results from a Hybrid Simulation of Variance Analysis of 12 Signals Simultaneously. Sampling Frequency: 10 Hz/Signal T = Measuring Time

KAN	MV		VAR
	T = 2 Min		
Ø	12	468	2696
1	25	976	12Ø2
2	5Ø	1953	682
3	1Ø1	3945	3Ø2
4	152	5937	76
5	2Ø2	789Ø	19
6	−12	−5Ø7	2737
7	−25	−1Ø15	122Ø
8	−50	−1992	686
9	−1ØØ	−3945	3ØØ
:	−152	−5976	77
;	−2Ø2	−7929	19
	T = 10 Min		
Ø	13	5Ø7	2427
1	25	976	1Ø82
2	51	1992	612
3	1Ø1	3945	271
4	152	5937	68
5	2Ø2	789Ø	17
6	−11	−468	2449
7	−24	−976	1Ø92
8	−5Ø	−1992	616
9	−1ØØ	−3945	270
:	−152	−5976	69
;	−2Ø2	−7929	17
	T = 20 Min		
Ø	12	468	2427
1	25	976	1Ø81
2	5Ø	1953	612
3	1Ø1	3945	271
4	152	5937	68
5	2Ø2	789Ø	17
6	−11	−468	2441
7	−25	−1Ø15	1Ø88
8	−5Ø	−1992	613
9	−1ØØ	−3945	268
:	−152	−5976	69
;	−2Ø2	−7929	17
	T = 30 Min		
Ø	13	5Ø7	243Ø
1	25	976	1Ø83
2	51	1992	612
3	1Ø1	3945	271
4	152	5937	68
5	2Ø2	789Ø	17
6	−11	−468	244Ø
7	−24	−976	1Ø87
8	−50	−1992	612
9	−1ØØ	−3945	268
:	−152	−5976	69
;	−2Ø2	−7929	17

Table 7 (c) and (d) shows the printout for the simulation of +5 and −5 V. The values of the accumulator were 200000 and 600000, respectively. The program for 8 signals, and thus 9-bit resolution, was used.

A result from a program run with 12 signals (9-bit resolution) is given in (e) of Table 7. This simulation is based on the same idea as Eqs. 7 to 9. However, this run uses negative values. The symbols ":" and ";" mean "10" and "11," respectively. The resulting printout routine was simplified by this trick [see the Teletype code (ASCII) in octal form].

Finally, Table 8 demonstrates the printout from a hybrid simulation. Twelve analog signals from the PACE computer were connected to the multiplexer of the PDP-9 computer.

Different means and variances were simulated. Statistical fluctuations were added to the signals to an analog Gaussian noise generator. The results of Table 8 show a linear dependence between the standard deviation (x) and the simulated noise amplitude.

One problem in the application of statistical measurements is the length of the measuring time. By this on-line method we can ask for the results at any time and stop the measurement, when the means and variances agree within given spread limits. Compare the variance printouts in Table 8 for the different periods of time. We see that the results for the measuring times of 20 and 30 min agree very closely. The means can be used to check the drift of the signals during a measurement.

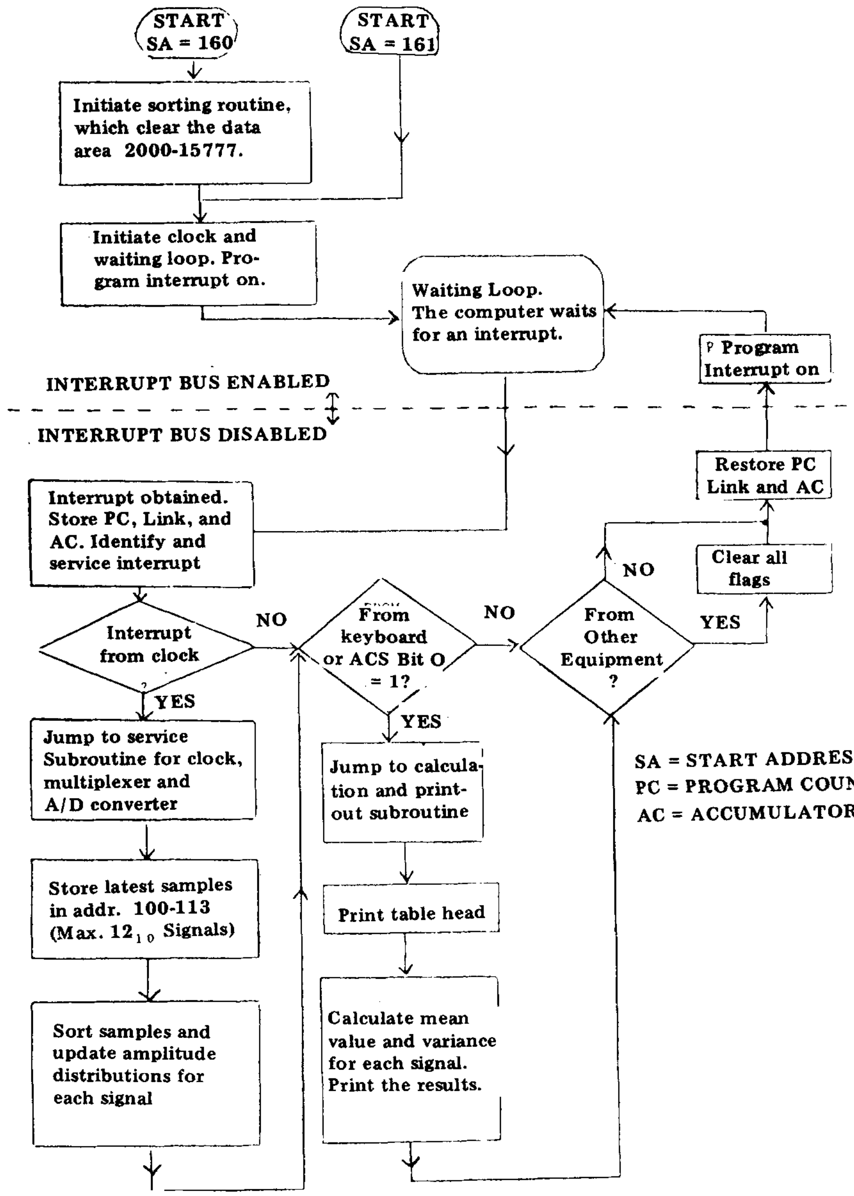

Fig. 5. Flow diagram of the on-line variance program.

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